Frieze Patterns

Topic
Geometric patterns in art

Problem
Friezes are repeating patterns. The problem is to recognize them, analyze them mathematically, and create some of your own.

Time
30 to 45 minutes

National Math Teaching Standards

ALGEBRA
- generalize patterns using explicitly defined and recursively defined functions

GEOMETRY
- analyze properties and determine attributes of two- and three-dimensional objects
- explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them
- understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices
- use various representations to help understand the effects of simple transformations and their compositions
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest, such as art and architecture

PROBLEM SOLVING
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts

- recognize and apply mathematics in contexts outside of mathematics

REPRESENTATION
- use representations to model and interpret physical, social, and mathematical phenomena

Materials
- graph paper
- calculators
- pencils

INTRODUCTION

A frieze is a repeating pattern, for example in dentil molding along a wall, in blankets, or pottery. Decorative railings, for example on balconies, often exhibit frieze patterns. As you move along, the pattern repeats itself, possibly after a flip. The idea of analyzing friezes mathematically is to think of physically moving the pattern until it lies on top of itself because of the repetition. The ways in which it can repeat are called transformations. There are several different types of such transformations (see Table 1 on page 3.02-3).

Now suppose we repeat a letter over and over again. For example **A**

AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA

Think of this as a frieze pattern. What frieze pattern is it? It can be reflected through any vertical line down the middle of an **A** or any vertical line separating two letters. Thus we label it as a **V** frieze.

Similarly
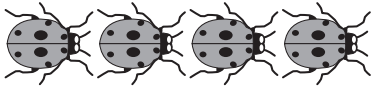
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

is labeled by **H**, and

OOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOO

by **R**. The frieze

⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓⧓

Table 1
Frieze transformations, with examples.

Type of Translation	Symbol	Example
horizontal translation	T	Translate left or right
reflection along a horizontal line	H	Reflect vertically in middle of ladybugs
reflection along a vertical line	V	Reflect across vertical line
180° rotation	R	Rotate 180° around a point
a glide reflection, a translation	G	Followed by a horizontal reflection, glide along and then reflect vertically

transforms by all three, so it can be labeled **VHR.** Some other possible labelings are **T, TR, TG, TV, THG, TRVG, TRHVG,** and maybe others.

PROCEDURE AND ANALYSIS

Q1. Find the transformations of the other letters. On the data sheet located on page 3.02-5, there are seven patterns. Find which (if any) of the above transformations give each pattern (fill in the chart). After this practice, construct some examples of your own frieze.

Examples

3.02.1

A frieze pattern. In order to classify it, we need
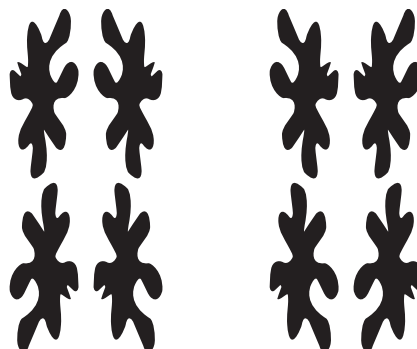only to look at one repeating block.
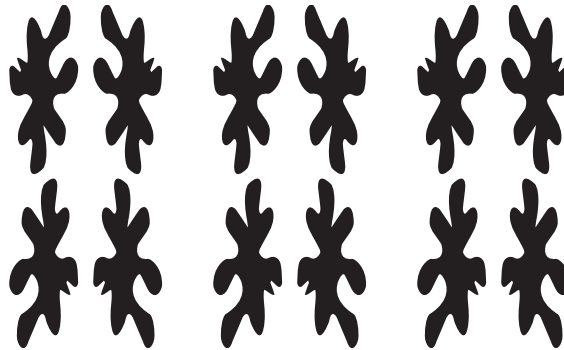
3.02.2

The pattern can be mapped by **T** to get the next block. It can also be
mapped onto itself by **R**, also by **H** and by **V**. We can also map the
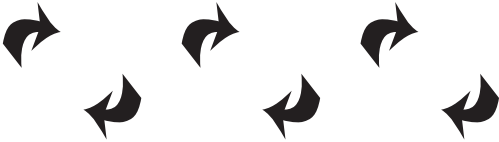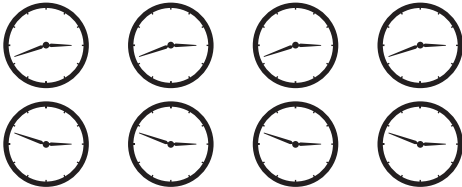first block over [missing figure] block by **G**.

3.02.3
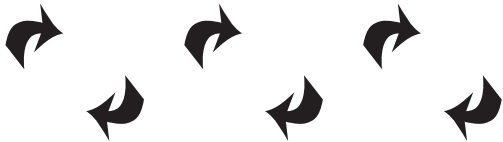
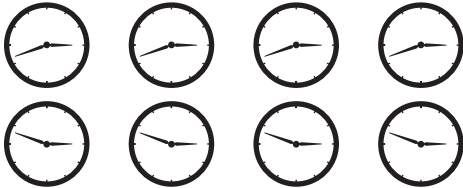This classifies as a **TRHVG** as these transformations all map the pattern to itself.

Q2. Frieze patterns can be compared to periodic functions. Consider the trigonometric functions. Use graphing calculators to graph the tan(x) function. Determine if this is a frieze pattern.

Data

	T	R	G	V	H	type

a
Typical Answers

	T	R	G	V	H	type
	y	y	n	n	n	TR
	y	y	y	y	n	TRVG
	y	n	n	n	n	T
	y	n	y	n	y	THG
	y	n	y	n	n	TG
	y	y	y	y	y	TRHVG
	y	n	n	y	n	TV

tangent